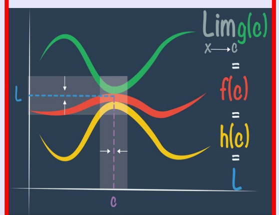
Calculus I

Lecture 34



Feb 19-8:47 AM

Given $f(x)=x^4-2x^2+3$ Y-Int $(0,3)$

1) Discuss Domain
Polynomial
$(-\infty,\infty)$

2) Find all intercepts
x-Int $f(x)=0$ $x^4-2x^2+3=0$
None $x^4-2x^2+1=-3+1$
$(x^2-1)^2=-2$
NO Real Solutions

3) Show that $f(x)$ is an even function.
$f(-x)=(-x)^4-2(-x)^2+3=x^4-2x^2+3=f(x)$
Symmetric w/t Y-axis

4) Find all $\underbrace{\text{Critical Points}}$.
$f'(x)=0$ or undefined

$f'(x)=4x^3-4x$ $4x^3-4x=0$ $4x(x^2-1)=0$
$4x(x+1)(x-1)=0$
$x=0$, $x=-1$, $x=1$
$(0,3)$, $(-1,2)$, $(1,2)$

5) Find all $\underbrace{\text{possible inflection Points}}$
$f''(x)=0$ or undefined

$f''(x)=12x^2-4$ $12x^2-4=0$ $x^2=\frac{1}{3}$ $x=\pm\frac{\sqrt{3}}{3}$
$\left(\frac{\sqrt{3}}{3},\frac{22}{9}\right)$, $\left(-\frac{\sqrt{3}}{3},\frac{22}{9}\right)$

$\left(\frac{\sqrt{3}}{3}\right)^4-2\left(\frac{\sqrt{3}}{3}\right)^2+3=$
$\left(\frac{1}{3}\right)^2-2\left(\frac{1}{3}\right)+3=$
$\frac{1}{9}-\frac{2}{3}+3=\frac{1-6+27}{9}=\frac{22}{9}$

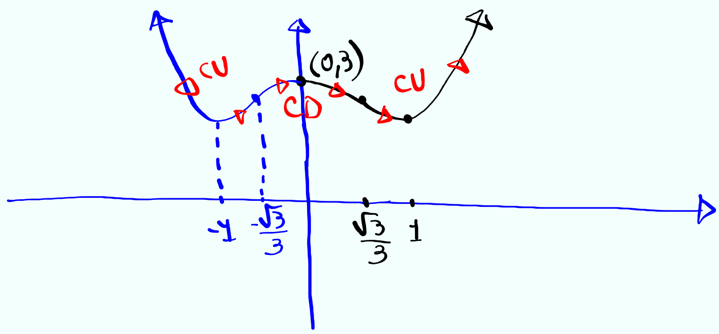
Oct 29-7:25 AM

$f'(x) = 4x(x+1)(x-1)$

$f''(x) = 12x^2 - 4$

Sign chart

x	$-\infty$		-1		$-\frac{\sqrt{3}}{3}$		0		$\frac{\sqrt{3}}{3}$		1		∞
$f'(x)$		$-$		$+$		$+$		$-$		$-$		$+$	
$f''(x)$		$+$		$+$		$-$		$-$		$+$		$+$	
$f(x)$		↘		↗		↗		↘		↘		↗	
			Min.		I.P.		Max		I.P.		Min.		



Oct 29-7:39 AM

Given $f(x) = \dfrac{x}{x^2+1}$

Y-Int $(0,0)$

1) Discuss domain

$x^2+1 \neq 0$

$(-\infty, \infty)$

2) Find all intercepts

x-Int. $f(x)=0$ $\quad \dfrac{x}{x^2+1} = 0$

$(0,0)$ $\quad x = 0$

3) Discuss asymptotes

No V.A.

$$\lim_{x\to\infty} \frac{x}{x^2+1} = 0$$

H.A. $y=0$

4) Show $f(x)$ is an odd function.

$$f(-x) = \frac{-x}{(-x)^2+1}$$

$$= -\frac{x}{x^2+1} = -f(x)$$

Since $f(-x) = -f(x)$, $f(x)$ is odd, symmetric w/t origin

Oct 29-7:49 AM

Find all critical Points

$$f(x) = \frac{x}{x^2+1} \qquad f'(x) = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$f'(x) = 0 \qquad 1 - x^2 = 0 \rightarrow x = 1 \qquad x = -1$

$(1, \frac{1}{2}) \qquad (-1, \frac{-1}{2})$

Find all Possible inflection Points

$$f''(x) = \frac{-2x(x^2+1)^2 - (1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4} = \frac{-2x(x^2+1) - 4x(1-x^2)}{(x^2+1)^3}$$

$$f''(x) = \frac{-2x(x^2+1+2(1-x^2))}{(x^2+1)^3} = \frac{-2x(3-x^2)}{(x^2+1)^3}$$

$$f''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}$$

P.I.P. $f''(x) = 0$ or und.

$x = 0 \qquad x = \sqrt{3} \qquad x = -\sqrt{3}$

$(0,0) \qquad (\sqrt{3}, \frac{\sqrt{3}}{4}), (-\sqrt{3}, \frac{-\sqrt{3}}{4})$

$$\frac{\sqrt{3}}{(\sqrt{3})^2+1} = \frac{\sqrt{3}}{4}$$

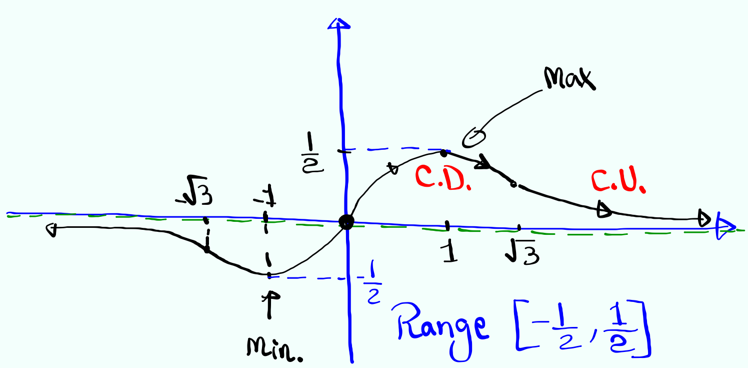
Oct 29-7:56 AM

$$f'(x) = \frac{(1+x)(1-x)}{(x^2+1)^2}$$

$$f''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}$$

Sign Chart

x	$-\infty$		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$		∞
$f'(x)$		$-$		$-$	•	$+$		$+$	•	$-$		$-$	
$f''(x)$		$-$	•	$+$		$+$	•	$-$		$-$	•	$+$	
$f(x)$		↘		↘	min	↗		↗	Max	↘		↘	



Range $[-\frac{1}{2}, \frac{1}{2}]$

Oct 29-8:07 AM

Find two numbers whose difference is 100 and their product is as small as possible.

Minimum

$\to x$ & y

$x - y = 100$

$y = x - 100$

$y = 50 - 100$

$= -50$

50 & -50

$xy = x(x-100)$

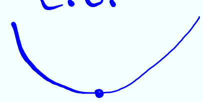
Let $f(x) = x(x-100)$

$f'(x) = 2x - 100$

C.P. $x = 50$

$f''(x) = 2 > 0$

C.U.



$\to (50, -50)$

Oct 29-8:18 AM

I have 100 m of fencing.

I want to build an enclosed rectangular shape for my dog to run around.

Find dimension to get max. area.

$2x + 2y = 100$

$x + y = 50$

$y = 50 - x$

$A = xy = x(50 - x)$

$f(x) = x(50 - x)$

$f(x) = 50x - x^2$

Max

$f'(x) = 50 - 2x$

$f'(x) = 0 \to x = 25$

$f''(x) = -2 < 0$

C.D.

Max

$(25, 25)$

Oct 29-8:25 AM

Looking ahead

$f'(x) = 5 \rightarrow f(x) = 5x + C$

$f'(x) = 2x - 4 \rightarrow f(x) = x^2 - 4x + C$

$f'(x) = \sec^2 x + 6 \rightarrow f(x) = \tan x + 6x + C$

$f'(x) = \cos x$

$f(0) = 2$

$\rightarrow f(x) = \sin x + C$

$f(0) = \sin 0 + C = 2$

$0 + C = 2$

$\boxed{C = 2}$

$\boxed{f(x) = \sin x + 2}$

Oct 29-8:32 AM